Math 2423
Spring 2016
Exam 1
2/19/16
Time Limit: 50 Minutes

Name (Print): ____________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- Partial credits apply. Part of an important step in mathematics is to write what you think in a coherent way, even when it does not yield fruitful results.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	20	
2	10	
3	35	
4	15	
5	20	
Total:	100	

1. (a) (5 points) Write out the definition of a definite integral $\int_a^b f(x)dx$ using Riemann sums. In addition, give a (brief) description of the symbols in your definition.

$$\int_a^b f(x)dx = \lim_{\substack{n\to\infty \\ (\text{or } \Delta x \to 0)}} \sum_{i=1}^{n} f(x_i)\Delta x \quad (3')$$

where x_i: (right) endpoint of the i^{th} subinterval

n: number of subintervals

Δx: length of each subinterval (or $\Delta x = \frac{b-a}{n}$) (2')

(b) (15 points) Using the definition, find $\int_0^2 x dx$ without evaluating the antiderivative. You may use the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

(2') $\Delta x = \frac{2}{n}$

(2') $x_i = \frac{2i}{n}$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\Delta x \quad (3')$$

$$= \sum_{i=1}^{n} \frac{2i}{n}\cdot\frac{2}{n} \quad (2')$$

$$= \frac{4}{n^2}\sum_{i=1}^{n} i \quad (3')$$

$$= \frac{4}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= \frac{4n+4}{2n}$$

$$\int_0^2 x dx = \lim_{n\to\infty} \frac{4n+4}{2n} \quad (1')$$

$$= \lim_{n\to\infty} \frac{4n+4}{2n}\cdot\frac{1/n}{1/n} \quad (2')$$

$$= \lim_{n\to\infty} \frac{4+4/n}{2} = \frac{4}{2} = 2$$

2. (10 points) Find $\frac{d}{dx}\int_0^{2x}\cos^3(t^2)\,dt$.

$$\frac{d}{dx}\int_0^{2x}\cos^3(t^2)\,dt = \cos^3\left((2x)^2\right)\frac{d}{dx}(2x) \quad (8')$$

$$= \cos^3(4x^2)\cdot 2 \quad (2')$$

(The $\frac{du}{dx}$ - notation for chain rule was used by some in the HW. It will receive full credit.)

3. (a) (14 points) Find $\int \sec^2(\sin x)\cos x\,dx$.

Let $u = \sin x$ (3')

$du = \cos x$ (2')

$$\int \sec^2(\sin x)\cos x\,dx$$

$$= \int \sec^2(u)\,du \quad (3')$$

$$= \tan(u) + C \quad (4')$$

$$= \tan(\sin x) + C \quad (2')$$

(b) (13 points) Find $\int_0^4 \sqrt{2x+1}\, dx$.

Let $u = 2x+1$ (3')
$du = 2dx$ (2')
$\frac{1}{2} du = dx$

when $x=0$, $u = 2\cdot 0 + 1 = 1$
$x=4$, $u = 2\cdot 4+1 = 9$ (2')

$$\int_0^4 \sqrt{2x+1}\, dx = \int_1^9 \sqrt{u}\cdot \frac{1}{2} du \quad (3')$$

$$= \frac{1}{2}\int_1^9 u^{1/2} du$$

$$= \frac{1}{2}\cdot\frac{2}{3} u^{3/2}\Big|_1^9 \quad (3')$$

$$= \frac{1}{3}\left(9^{3/2} - 1^{3/2}\right) = \frac{1}{3}(27-1) = \frac{26}{3}$$

(c) (8 points) Find $\int \frac{\cos(x^{-2})}{x^3}\, dx$

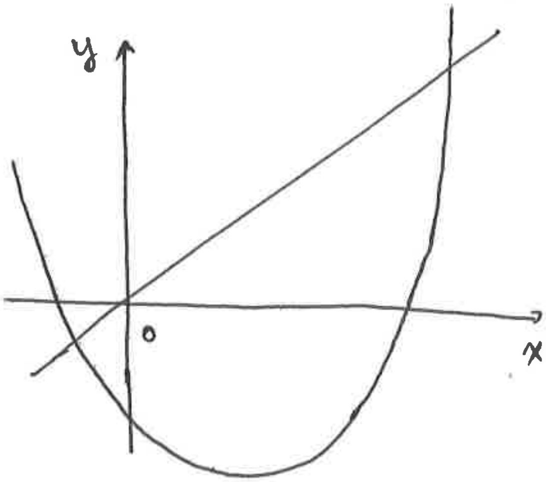
Let $u = x^{-2}$ (3')
$du = -2x^{-3} dx$ (2')
$-\frac{1}{2} du = \frac{1}{x^3} dx$

$$\int \frac{\cos(x^{-2})}{x^3} dx = -\frac{1}{2}\int \cos(u)\cdot du \quad (3')$$

$$= -\frac{1}{2}\sin(u) + C \quad (1')$$

$$= -\frac{1}{2}\sin(x^{-2}) + C$$

4. (15 points) Find the area of the region enclosed by the graphs of $y = x^2 - 4x - 6$ and $y = x$.



$x^2 - 4x - 6 = x$ (3')

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6, \quad x = -1$ (3')

$\int_{-1}^{6} (x - (x^2 - 4x - 6))\, dx$ (4')

$= \int_{-1}^{6} (x - x^2 + 4x + 6)\, dx$

$= \int_{-1}^{6} (-x^2 + 5x + 6)\, dx$ (1')

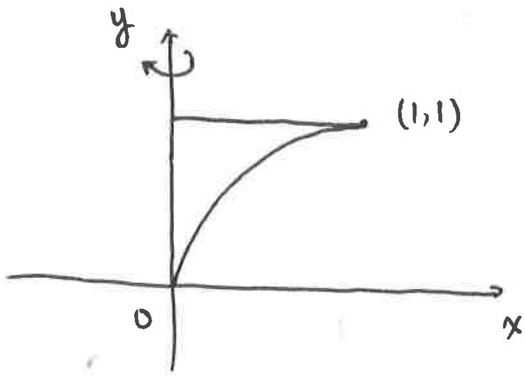
$= \left(-\frac{x^3}{3} + \frac{5x^2}{2} + 6x\right)\Big|_{-1}^{6}$ (3')

$= -\frac{216}{3} + \frac{5 \cdot 36}{2} + 36 - \left(-\frac{-1}{3} + \frac{5}{2} - 6\right)$ (1')

$= -72 + 90 + 36 - \frac{1}{3} - \frac{5}{2} + 6$

$= 60 - \frac{1}{3} - \frac{5}{2} = \frac{360 - 2 - 15}{6} = \frac{343}{6}$

5. (a) (17 points) Find the volume of the solid by revolving the region enclosed by $y = 1$, $y = \sqrt{x}$ and the y-axis around the y-axis.



$$\int_0^1 (\text{area of the disc}) \cdot dy$$

$$= \int_0^1 \pi (\text{radius})^2 dy \quad (4')$$

$$= \pi \int_0^1 (y^2)^2 dy \quad (3')$$

$$= \pi \int_0^1 y^4 dy \quad (1')$$

$$= \frac{1}{5}\pi y^5 \Big|_0^1 \quad (1')$$

$$= \frac{\pi}{5}$$

$y = \sqrt{x}$

$y^2 = x$ (3')

(Alternatively, using the "label" or "cylindrical shell (same method with the name given in the textbook)" method correctly will receive full credits.)

(b) (3 points) State the name of the method you use(you can either use the name we came up in class or the one used in the textbook.)

The Disc Method